

# Limits on excited $\tau$ leptons masses from leptonic $\tau$ decays

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## Abstract

We study the effects induced by excited leptons on the leptonic  $\tau$  decay at one loop level. Using a general effective lagrangian approach to describe the couplings of the excited leptons, we compute their contributions to the leptonic decays and use the current experimental values of the branching ratios to put limits on the mass of excited states and the substructure scale.

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The values for the leptonic  $\tau$  decays [1] have confirmed the validity of the standard model (SM) [2] as the theory of electroweak interactions at the current scales of energies. Moreover, the results have reached such precision that they have opened the possibility to constrain significantly some physics beyond the standard model, for instance, compositeness [3,4].

The family structure of the known fermions, among other regularities, has been considered as an indication to expect that the SM fermions and perhaps massive gauge bosons possess some kind of substructure. The idea of composite models assumes the existence of an underlying structure, characterized by scale  $\Lambda$ , with the fermions sharing some of the constituents. As a consequence, excited states of each known lepton should show up at some energy scale, and the SM should be seen as the low-energy limit of a more fundamental theory.

Precise measurements of anomalous magnetic moment of muon and electron indicate that first and second family of leptons are elementary particles with high grade of precision. In this conditions, for simplicity, we take an conservative point of view and we assume that only  $\tau$ ,  $\nu_\tau$  can be composite. They are largely most massive than the others leptons and their properties are less known. Then, in this work, we consider  $\tau$  and  $\nu_\tau$  leptons as composite and we keep the other leptons as elementary. In this conditions we only consider excited states of leptons  $\tau$  and  $\nu_\tau$ . It is our fundamental hypothesis that can be understood considering either that the first and second family are elementary or their associated substructure scale are much bigger than the  $\tau$  compositeness scale ( $\Lambda_e, \Lambda_\mu \gg \Lambda_\tau$ ).

We still do not have a satisfactory model, able to reproduce the whole particle spectrum. Due to the lack of a predictive theory we should rely on a model-independent approach to explore the possible effects of compositeness, employing effective Lagrangian techniques to describe the couplings of these states.

Several experimental collaborations have been searching for excited states [5], in particular on  $\tau^*$  and  $\nu_\tau^*$ . Their analyses are based on an effective  $SU(2) \otimes U(1)$  invariant Lagrangian,

proposed some years ago by Hagiwara et. al. [6]. Also a series of phenomenological studies of excited fermions have been carried out in several experiments. Moreover theoretical bounds have been derived from the contribution to the anomalous magnetic moment of leptons and the Z scale observables at LEP.

On the other hand, an important source of indirect information about new particles and interactions is the precise measurement of the leptonic branching ratios (BR) of lepton  $\tau$  [4]. Virtual effects of these new states can modify the SM predictions for the BR, and the comparison with the experimental data can impose bounds on their masses and couplings.

In this paper we use a general effective Lagrangian approach [6] to investigate the effects induced by excited tau and tau neutrinos in the leptonic branching ratios at one loop level. We show our results as an allowed region in the  $(m^*, f/\Lambda)$  plane. We find bounds for the substructure scale as a function of excited mass and compare them with bounds obtained for different experiments, in particular OPAL [5], bounds coming from the anomalous weak-magnetic moment of the tau lepton [7,8] and precision measurement on the Z peak [9].

The SM prediction for the leptonic decay width, including electroweak radiative corrections, is

$$\Gamma_{SM}(\tau \rightarrow l\nu_l\nu_\tau) = \frac{G_F^2 m_\tau^5}{192\pi^3} f(m_l^2/m_\tau^2) r \quad (1)$$

where

$$f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \ln(x)$$

and the factor  $r$  takes into account the radiative corrections that are not absorbed in the Fermi constant  $G_F$ , and is estimated to be 0.9960.

The leptonic branching can be calculated in terms of the observed  $\tau$  lifetimes,  $\tau = (290.0 \pm 1.2) \times 10^{-15}$  s, and the measured value for the mass of the  $\tau$  lepton,  $m_\tau = 1777.0 \pm 0.28$  MeV. With these, it is possible to stimate the theoretical values for the branching ratio of the electronic and the muonic modes:  $B_e^{th} = 0.1777 \pm 0.0007$  and  $B_\mu^{th} = 0.1728 \pm 0.0008$ ,

respectively which are now in good agreement with the experimental results:  $B_e^{exp} = 0.1781 \pm 0.0007$  and  $B_\mu^{exp} = 0.1737 \pm 0.0009$ , once the theoretical uncertainties are properly taken into account [1]. We have also considered the quantity  $R_\tau$ , which is defined as

$$R_\tau = \frac{\Gamma_\tau - \Gamma_e - \Gamma_\mu}{\Gamma_e}. \quad (2)$$

Using the measured values of the leptonic branching ratios, one finds the value

$$R_\tau^{exp} = \frac{1 - B_e - B_\mu}{B_e} = 3.64 \pm 0.019 \quad (3)$$

In order to study limits on the scale of compositeness, we shall consider the contribution to the decay width, due to indirect effects induced by excited  $\tau^*$  and  $\nu_\tau^*$  at one loop level. For hypothesis the other leptons are considered either elementary or with their excited states decoupled due to much bigger compositeness scale. We consider excited fermionic states with spin and isospin  $\frac{1}{2}$ , and we assume that the excited fermions acquire masses before the  $SU(2) \times U(1)$  breaking, so that both left-handed and right-handed states belong to weak isodoublets (vector-like model). The effective dimension five Lagrangian that describes the coupling of excited-usual fermions, which is  $SU(2) \times U(1)$ , can be written as [8]

$$\begin{aligned} \mathcal{L}_{eff} = & - \sum_{V=\gamma, Z, W} T_{VLl} \bar{L} \sigma^{\mu\nu} P_L l \partial_\mu V_\nu \\ & - i \sum_{V=\gamma, Z} Q_{VLl} \bar{L} \sigma^{\mu\nu} P_L l W_\mu V_\nu + h.c. \end{aligned} \quad (4)$$

where  $L = \nu_\tau^*, \tau^*$  represent the excited states, and  $l = \nu_\tau, \tau$ , the usual light fermions of third generation. A pure left-handed structure is assumed for these couplings. The coupling constants  $T_{VLl}$  are given by

$$\begin{aligned} T_{\gamma\tau^*\tau} &= -\frac{e}{2\Lambda}(f + f') \\ T_{\gamma\nu^*\nu} &= \frac{e}{2\Lambda}(f' - f) \\ T_{Z\tau^*\tau} &= -\frac{e}{2\Lambda}(f' \cot \theta_W - f \tan \theta_W) \\ T_{Z\nu^*\nu} &= \frac{e}{2\Lambda}(f' \cot \theta_W + f \tan \theta_W) \\ T_{W\tau^*\nu} &= T_{W\nu^*\tau} = \frac{e}{\sqrt{2} \sin \theta_W \Lambda} f', \end{aligned} \quad (5)$$

where  $\Lambda$  is the compositeness scale,  $f'$  and  $f$  are weight factors associated to the  $SU(2)$  and  $U(1)$  coupling constants and  $\theta_W$  is the weak mixing angle. The quartic interaction couplings  $Q_{VLI}$  are given by,

$$\begin{aligned} Q_{\gamma\tau^*\nu} &= -Q_{\gamma\nu^*\tau} = -\frac{e^2\sqrt{2}}{2\sin\theta_W\Lambda}f', \\ Q_{Z\tau^*\nu} &= -Q_{Z\nu^*\tau} = -\frac{e^2\sqrt{2}\cos\theta_W}{2\sin^2\theta_W\Lambda}f'. \end{aligned} \quad (6)$$

The coupling of gauge bosons to excited leptons in a vector-like model are given by the following renormalizable lagrangian (dimension four),

$$\mathcal{L}_{ren} = - \sum_{\gamma,Z,W} A_{VLL} \bar{L} \gamma^\mu V_\mu L \quad (7)$$

which is  $SU(2) \times U(1)$  invariant. The coupling constants are given by,

$$\begin{aligned} A_{\gamma\tau^*\tau^*} &= -e, \\ A_{\gamma\nu^*\nu^*} &= 0, \\ A_{Z\tau^*\tau^*} &= \frac{(2\sin^2\theta_W - 1)e}{2\sin\theta_W\cos\theta_W}, \\ A_{Z\nu^*\nu^*} &= \frac{e}{2\sin\theta_W\cos\theta_W}, \\ A_{W\tau^*\nu^*} &= \frac{e}{\sqrt{2}\sin\theta_W}. \end{aligned} \quad (8)$$

The contributions of the excited leptons to amplitude for the leptonic tau decay at one-loop level are represented in figure 1. We consider the tree level amplitude plus the legs, box and vertex corrections which are represented in figure 2 and 3, respectively. The dominant contributions from this radiative corrections are given by the interference between the SM term and the new contributions from excited  $\tau^*$  and  $\nu_\tau^*$ . We find that the decay width can be written as

$$\Gamma = \Gamma_{SM}(1 + \delta\Gamma^{(RC)}) \quad (9)$$

where the SM part is given by Eq(1), and the expression for the new part, taking  $m_\tau^* = m_{\nu_\tau}^* = m^*$ , is

$$\delta\Gamma^{(RC)} = \frac{\alpha}{f(m_l^2/m_\tau^2)r} \left( \frac{m^* f}{\Lambda} \right)^2 (\mathcal{L} + \mathcal{V} + \mathcal{B}). \quad (10)$$

where  $\alpha$  is the fine structure constant. The functions  $\mathcal{L}$ ,  $\mathcal{V}$  and  $\mathcal{B}$  correspond to the interference between the SM term and the Leg, Vertex and Box radiative corrections, respectively.

The functions  $\mathcal{L}$ ,  $\mathcal{V}$  and  $\mathcal{B}$  are given by

$$\begin{aligned} \mathcal{L} = & -\frac{3}{4\pi} \left[ -\frac{c_w^6 \xi_z^6 \ln(c_w^2 \xi_z)}{s_w(1 - c_w^2 \xi_z)^2} - \frac{(c_w^4 + s_w^4) \xi_z^3 \log(\xi_z)}{2s_w^2 c_w^2 (1 - \xi_z)^2} + \frac{11}{3} \right. \\ & + \left( 2 + \frac{2 + c_w^2 \xi_z}{s_w^2} + \frac{(2 + \xi_z)(c_w^4 + s_w^4)}{2c_w^2 s_w^2} \right) \ln \xi_\Lambda \\ & \left. + \frac{(c_w^4 + s_w^4)(22 - 11\xi_z - 17\xi_z^2)}{12c_w^2 s_w^2 (1 - \xi_z)} + \frac{(22 - 11c_w^2 \xi_z - 17c_w^4 \xi_z^2)}{6s_w^2 (1 - c_w^2 \xi_z)} \right], \end{aligned} \quad (11)$$

$$\begin{aligned} \mathcal{V} = & \frac{1}{16\pi s_w^2 \xi_z (1 - \xi_z)^2 (1 - c_w^2 \xi_z)^2} \left[ -24c_w^4 \xi_z^3 (1 - \xi_z)^2 \ln(c_w^2 \xi_z) \right. \\ & + 6(1 - c_w^2 \xi_z) \xi_z^3 \left( 4(1 - \xi_z) + s_w^2 \xi_z \left( \frac{c_w^2}{s_w^2} - \frac{s_w^2}{c_w^2} + \frac{4}{c_w} \right) \right) \ln(\xi_z) \\ & + 6(1 - c_w^2 \xi_z) s_w^2 \xi_z (1 - \xi_z)^2 \left( (2 + \xi_z) \left( 4 - \frac{2}{c_w} - \frac{c_w^2}{s_w^2} + \frac{s_w^2}{c_w^2} \right) \right. \\ & \left. + 8 \left( \frac{1}{s_w} - 1 \right) \right) \ln(\xi_\Lambda) + (1 - c_w^2 \xi_z)(1 - \xi_z) s^2 \xi_z (20\xi_z(1 - \xi_z) \\ & \left. + \frac{12}{c_w} \xi_z(1 + \xi_z) + \left( \frac{c_w^2}{s_w^2} - \frac{s_w^2}{c_w^2} \right) (2 + 5\xi_z - \xi_z^2) \right) \right], \end{aligned} \quad (12)$$

$$\begin{aligned} \mathcal{B} = & \frac{9(1 - 3s_w^2) \xi_z}{8\pi s_w^2} \left[ \frac{1}{36s_w^2 (1 - \xi_z)(1 - c_w^2 \xi_z)} \left( 11(1 - \xi_z)(1 - c_w^2(1 + \xi_z)) \right. \right. \\ & \left. \left. + c_w^4 \xi_z \right) + 6\xi_z(1 - c_w^2 \xi_z - c_w^4(1 - \xi_z)) \ln(\xi_z) \right] \end{aligned} \quad (13)$$

$$- 6c_w^4 \ln(c_w^2) \xi_z (1 - \xi_z) \left[ \frac{\ln(\xi_\Lambda)}{6} - \frac{43 - 26c_w^2}{72(1 - 3s_w^2)} \right]$$

where  $\xi_z = (m_Z/m^*)^2$ ,  $\xi_w = (m_W/m^*)^2$ ,  $\xi_\Lambda = (m_\Lambda/m^*)^2$ ,  $s_w = \sin \theta_w$  and  $c_w = \cos \theta_w$ . In the following analysis we take  $m_\tau^* = m_{\nu_\tau}^* = m^*$  and  $f = f'$ .

The loops contributions of the excited leptons were evaluated in  $D = 4 - 2\epsilon$  dimensions using the dimension regularization method, which is a gauge-invariant regularization procedure, where the pole at  $D = 4$  is identified with  $\ln \Lambda^2$ .

We should notice that since we are including non-renormalizable operators the results of the loops are, in principle, quadratically divergent with the scale  $\Lambda$ . However, since we are restricting ourselves to  $SU(2) \times U(1)$  gauge invariant operators, the final results for the physical observables are, at most, logarithmically divergent. In other words, all quadratic (or higher) dependence on  $\Lambda$  is simply cancelled by counter terms coming from the high-energy theory. At one loop at best only a logarithmic dependence on the scale of new physics can be extracted purely from the low-energy effective lagrangian.

We evaluate the renormalization constants by imposing the on-shell renormalization conditions on the renormalized transition amplitudes. In this scheme we compute the diagrams of the external legs to obtain the renormalization of the lepton wave functions taking the mass of the particles as the experimental value.

To obtain bounds for the excited mass and the substructure scale we compare our theoretical results:  $B_e^{th} = \Gamma_e^{th}/\Gamma_\tau$  (where  $\Gamma_e^{th}$  is given by eq.(9)) with the experimental value of  $B_e^{ex}$ ,  $B_\mu^{ex}$  and  $R_\tau$ . We find that the most restrictive bound come from  $B_e^{ex}$  and then we use it to put limits on  $\Lambda$  and  $m^*$ . We consider  $B_e^{th}$  as a function of  $m^*$  and  $\Lambda$  and then the limits are obtained by comparing  $B_e^{th}(m^*, \Lambda)$  with the experimental value of  $B_e^{ex}$ .

In Fig. 4 we show our bounds showing the allowed regions for the excited lepton masses and the ratio  $f/\Lambda$  at 95% C.L. The curve that limit the region is obtained intersecting the function  $B_e^{th}(m^*, \Lambda)$  with the experimental value  $B_e^{ex} \pm \Delta B_e$ . It is understood that the

non-allowed region sets co-related bounds for the excited lepton mass and  $\Lambda$ . To compare our results with other bounds we include in this figure the results from OPAL collaboration and bounds coming from the weak-magnetic moment of the  $\tau$  lepton on the Z peak [5,7,8]. Moreover in figure 5 we include a comparison between ours results and bounds coming from precision measurements on the Z peak [9]. It is important observer that the bounds from the leptonic tau decay is safe in the  $\Lambda > m^*$  region where the decoupling of new physics work.

Finally we study the decoupling properties of the new contributions which cancel out in the limit of large substructure scale and fixed excited mass. The results are shown in figure 6. The curves represent the variation of the new contributions ( $\delta\Gamma^{RC}$ ) with  $\Lambda$  for different values of  $m^*$ .

Summarizing, we have considered the possibility that the lepton tau and their neutrino have some kind of substructure. We have modeled the interactions involving their excited states through a renormalizable lagrangian (vector-like model) and an effective dimension-5 operator that couples ordinary particles with excited particles and gauge bosons and we have considered that the other leptons are either elementary or their compositeness scale are much bigger than the tauonic one. By computing the contributions of these interactions to the radiative corrections of leptonic tau decay and by comparing them with the well mesured branching ratios, we have obtained bounds on the excited state masses and the compositeness scale  $\Lambda$ . This bounds are most restrictives that others obtained from direct productions [5], from radiative corrections to weak-magnetic moment of the  $\tau$  lepton on the Z peak [7,8] and from precision measurements on the Z peak [9].

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## REFERENCES

- [1] Particle Data Group, C. Caso *et al.*, The European Physical Journal C3, 1 (1998).
- [2] S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967); A. Salam, *Elementary Particle Theory: Relativistic Groups and Analyticity (novel Symposium No. 8)*, edited by N. Svartholm (*Almqvist and Wiksell, Stockholm, 1968*), p.367; S.L.Glashow, *Nucl. Phys.* 22, 579 (1961).
- [3] For a Review, see for instance: H. Harari, Phys. Reports 104 (1984) 159; H.Terazawa, Proceedings of the XXII International Conference on High Energy Physics, Leipzig, 1984, edited by A. Meyer and E. Wieczorek, p 63; W. Buchmuller, *Acta Phys. Austriaca*, Suppl. XXVII (1985) 517; M.E. Peskin, in *Proceedings of the International Symposium on Lepton and Photon Interaction at High Energies, Kyoto, Japan, 1985*, edited by M. Konuma and K.Takahashi (*RIPF, Kyoto, 1986*), p. 714.
- [4] J.L. Diaz Cruz and O.A. Sampayo, Phys. Rev. D49, R2149 (1994)
- [5] L3 Collabotation, M. Acciarri *et al.*, Phys. Lett. B401, 139 (1997). DELPHI Collaboration, P. Abreu *et al.*, Phys. Lett. B393, 245 (1997). OPAL Collaboration, K. Acherstaff *et al.*, Phys. Lett. B391, 197 (1997).
- [6] K. Hagiwara, S. Komamiya and D. Zeppenfeld, Z. Phys. C29, 115, (1985).
- [7] J. Bernabeu, G.A. Gonzalez-Sprinber, M. Tung and J.Vidal, Nucl. Phys. B436, 474 (1995).
- [8] M.C. Gonzalez-Garcia and S.F. Novaes, Phys. Lett. B389, 707 (1996).
- [9] M.C. Gonzalez-Garcia and S.F. Novaes, Nuclear Physics B486, 3, 1997.

## Figure Captions

**Figure 1:** Diagramatic representation for the contribution of the leptonic tau decay amplitude. Box and dashed blobs represent the contributions of excited tau and tau neutrino.

**Figure 2:** Self-energy and box corrections contributing to leptonic tau decay.

**Figure 3:** Vertex corrections contributing to leptonic tau decay.

**Figure 4:** Dashed zone represent the allowed region at 95% C.L. The curves represent bounds coming from (a) leptonic tau decay, (b) single production at OPAL and (c) weak-magnetic moment of tau lepton.

**Figure 5:** Excluded regions in the  $\Lambda$  versus  $m^*$  plane (below of the curves), at 95 % C.L., from (a) leptonic tau decay and (b) precision measurements on the Z peak.

**Figure 6:** Decoupling properties of the new contributions as a function of  $\Lambda$  for different values of  $m^*$ .

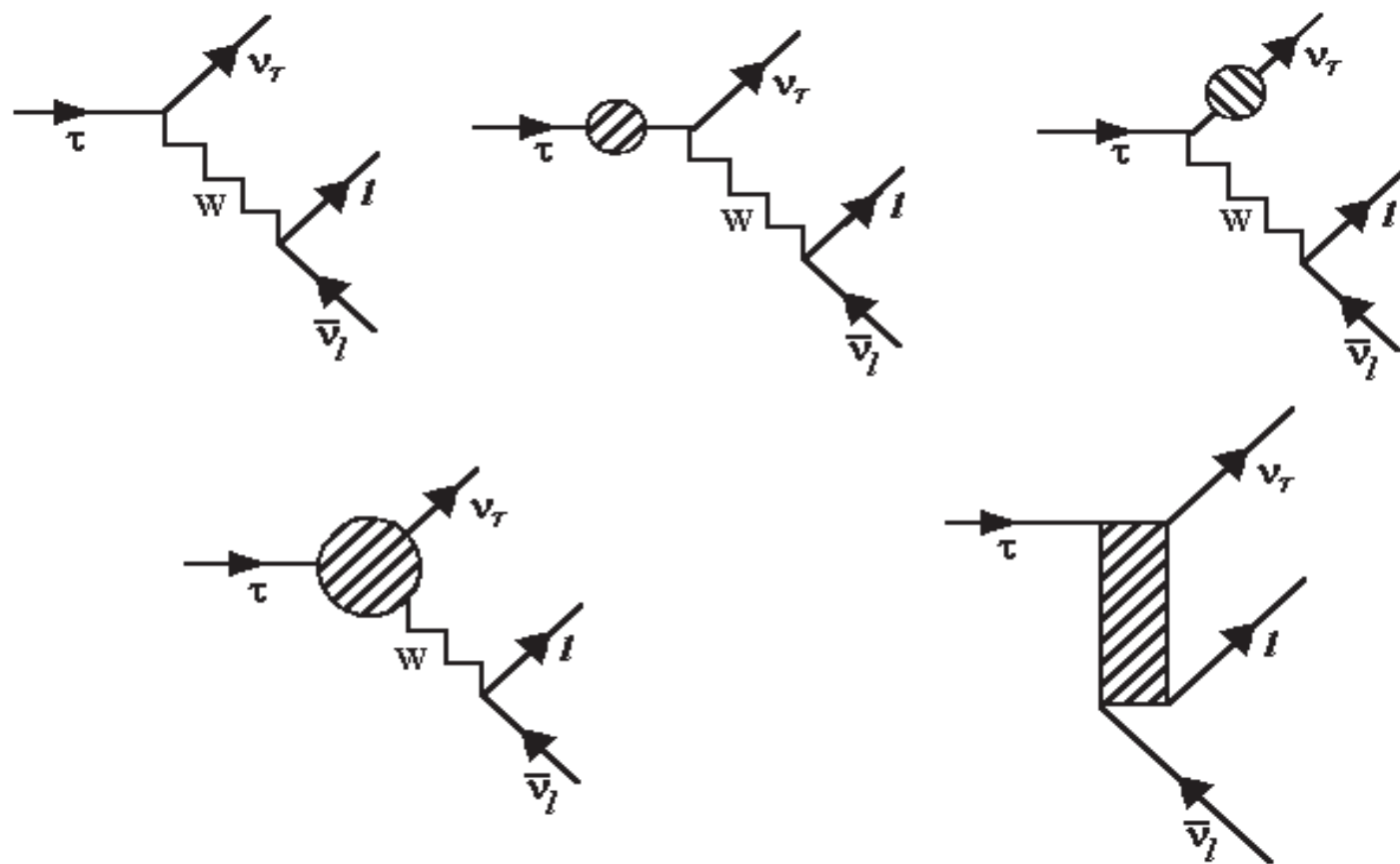


Fig. 1

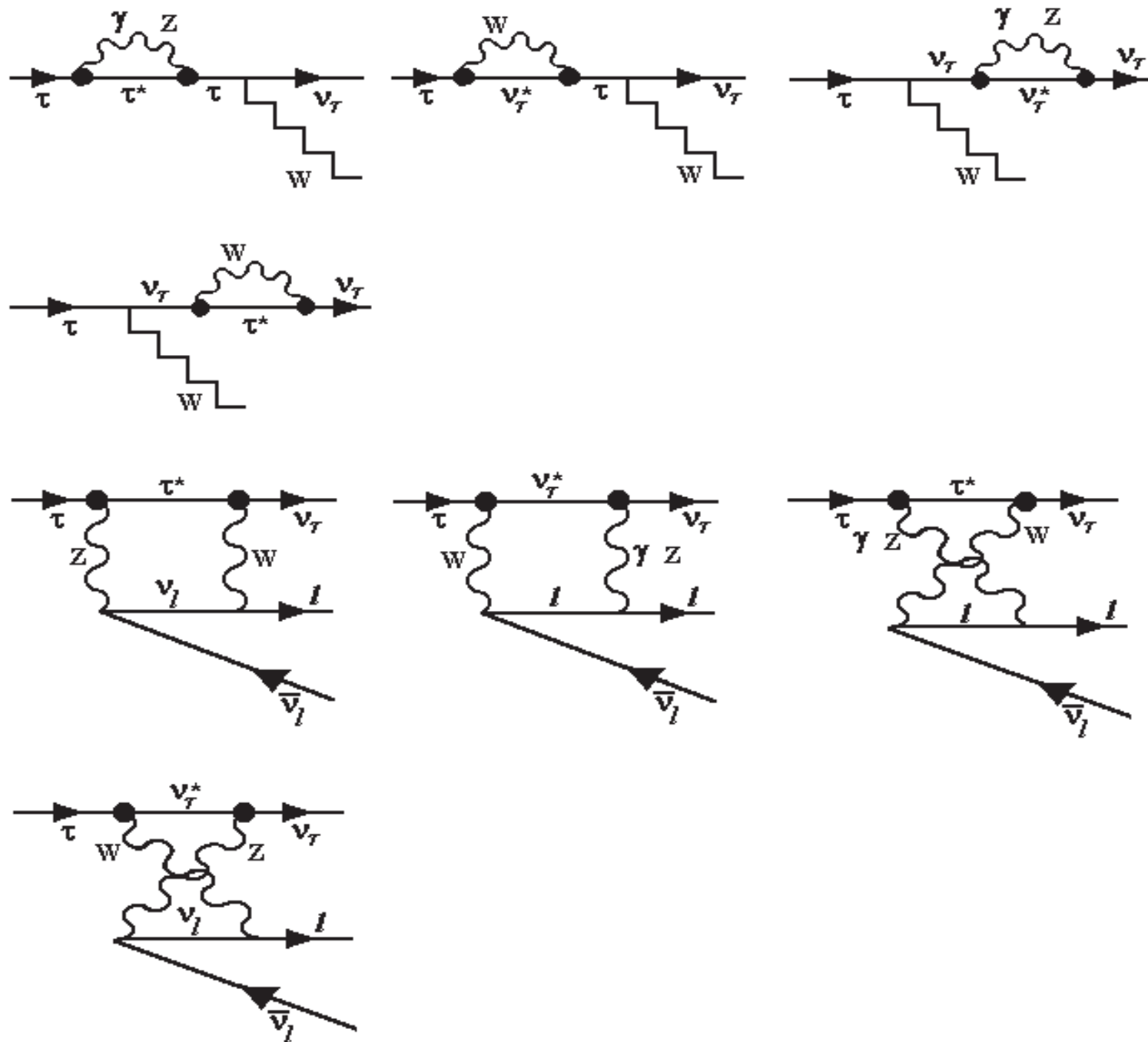


Fig. 2

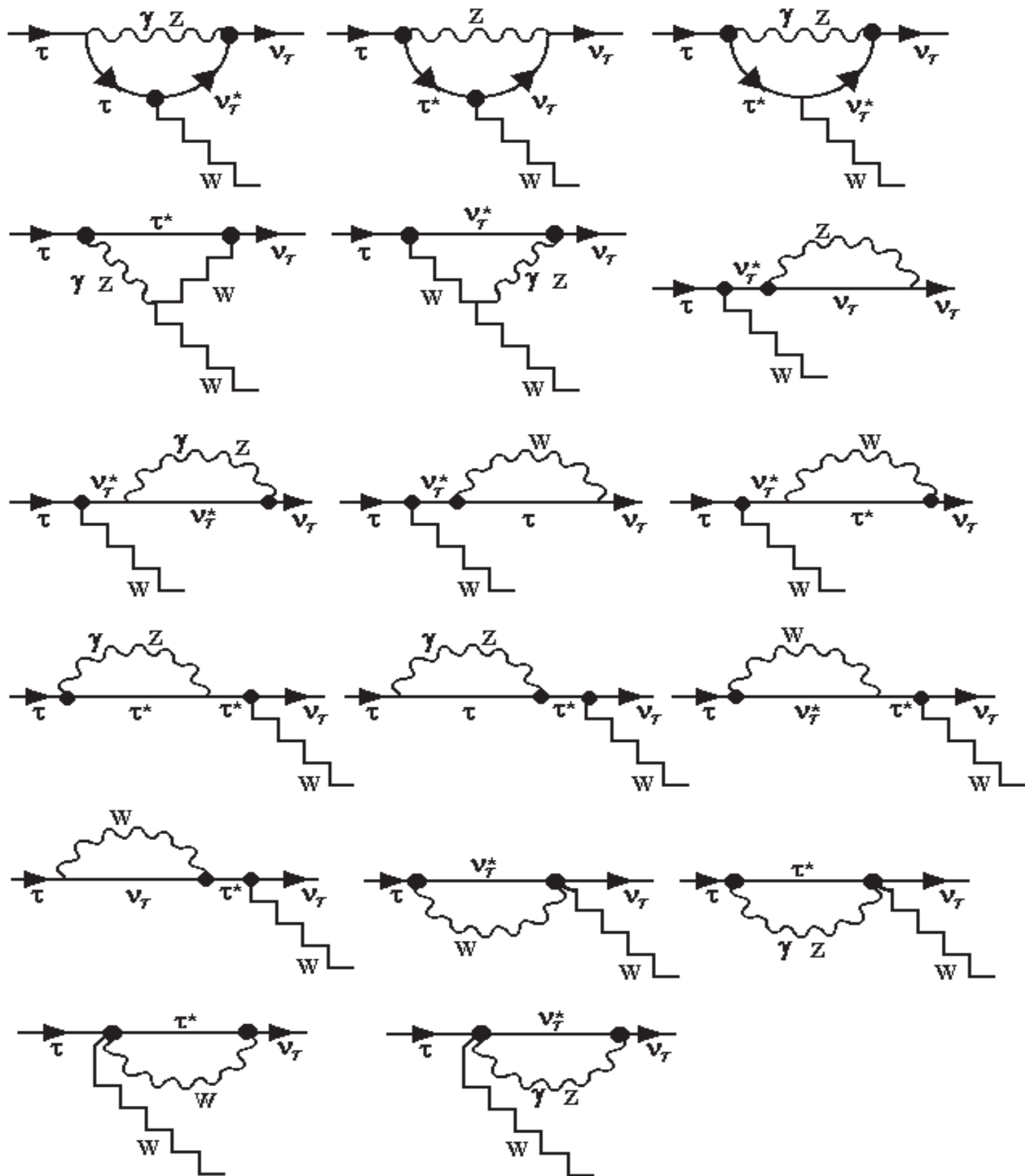


Fig. 3

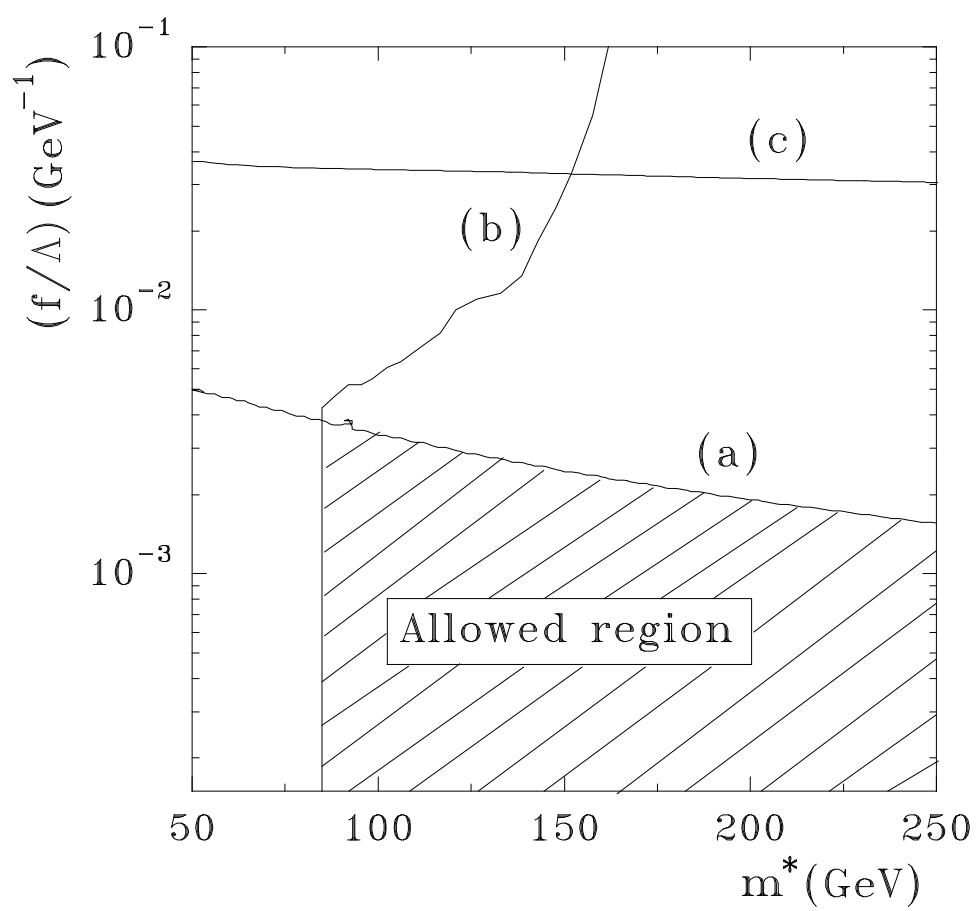


Fig. 4

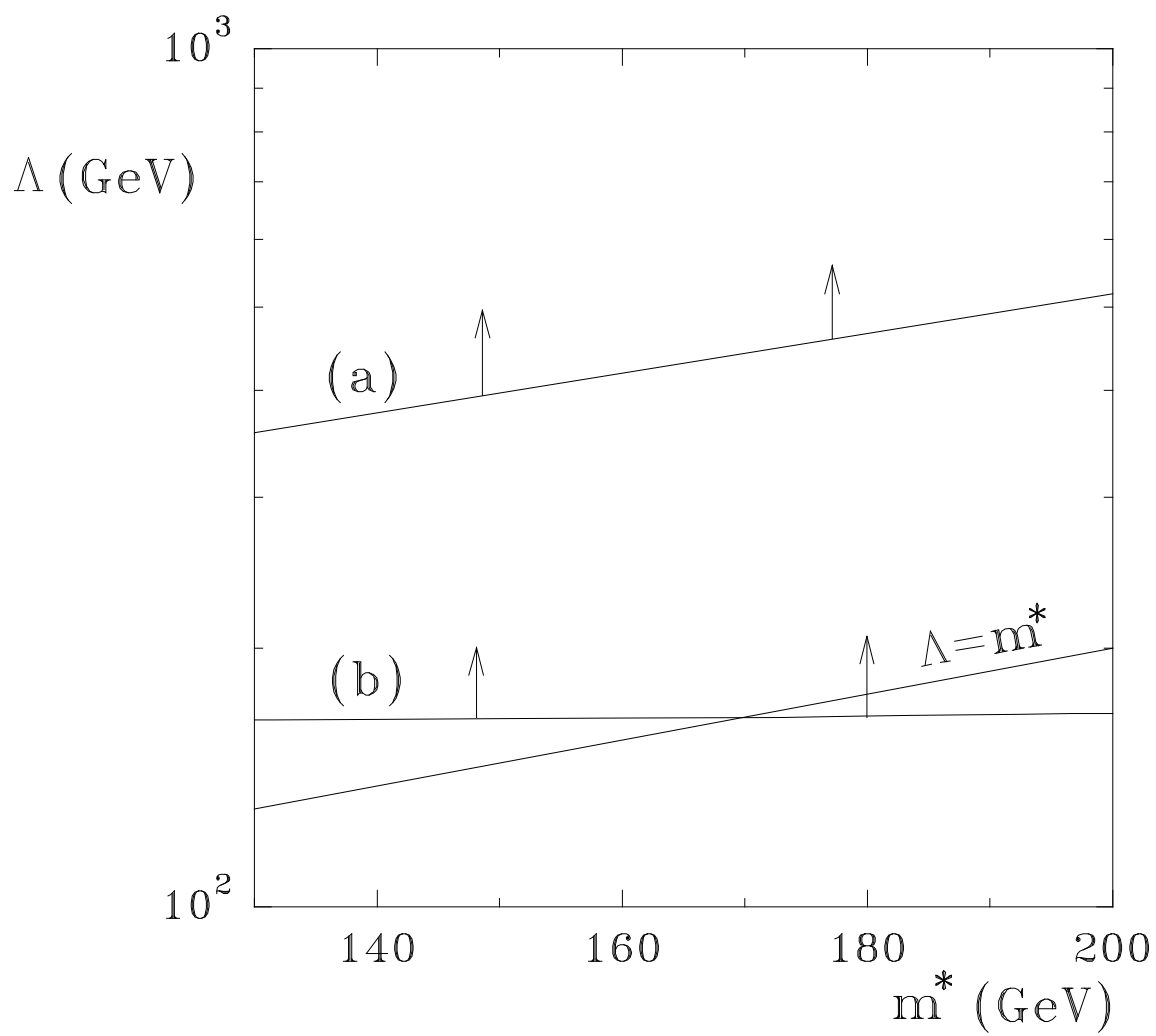


Fig. 5

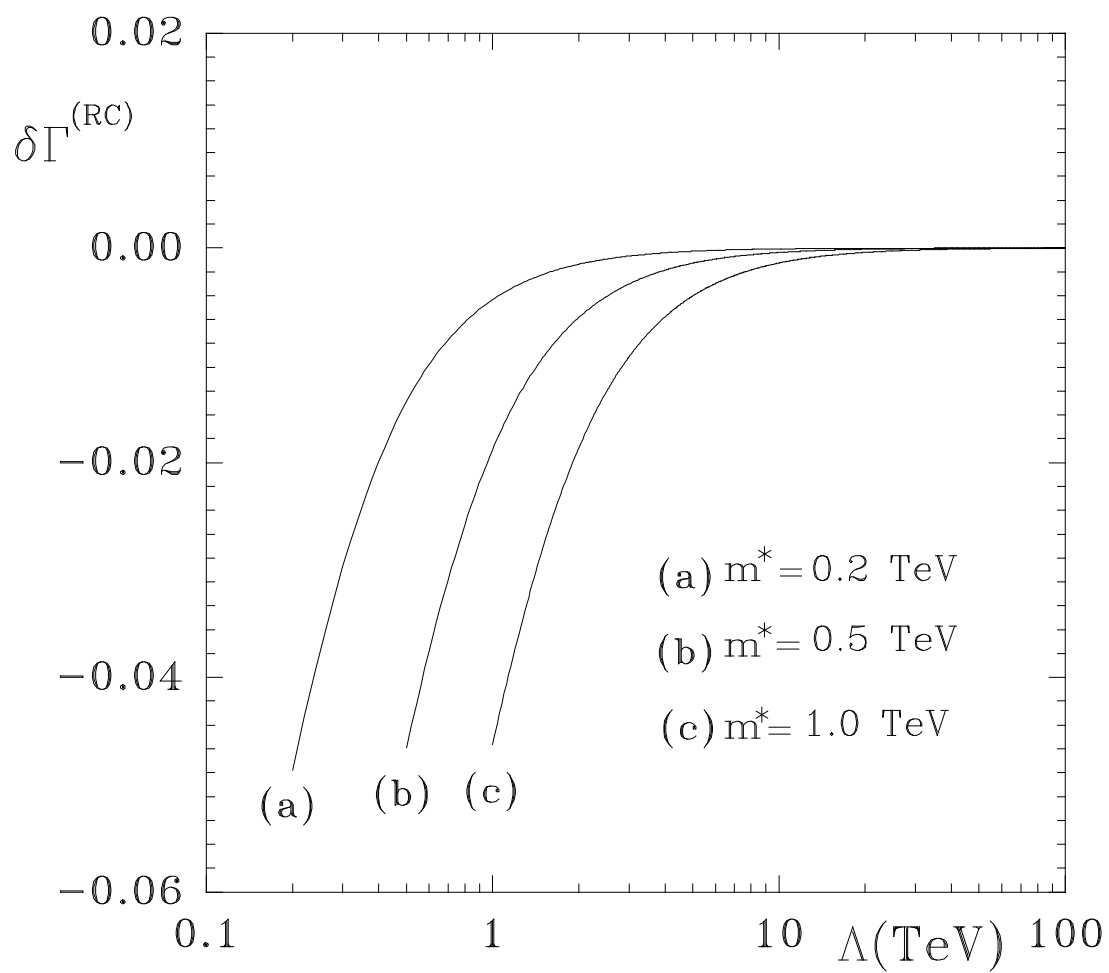


Fig. 6